IFUP-TH 8/97

On the evaluation of universal non-perturbative constants in $O(N)$ σ models.

Massimo Campostrini, Andrea Pelissetto, Paolo Rossi, and Ettore Vicari
Dipartimento di Fisica dell'Università and I.N.F.N.,I-56126 Pisa, Italy

(February 1, 2008)

Abstract

We investigate the relation between on-shell and zero-momentum non-perturbative quantities entering the parametrization of the two-point Green's function of two-dimensional non-linear $O(N)$ σ models. We present accurate estimates of ratios of mass-scales and renormalization constants, obtained by an analysis of the strong-coupling expansion of the two-point Green's function. These ratios allow to connect the exact on-shell results of Refs. [1,2] with typical zero-momentum lattice evaluations. Our results are supported by the $1/N$-expansion.

PACS numbers: 11.10.-z, 11.15.Me, 11.15.Pg, 75.10.Hk

Introduction. Physical quantities which are independent of coordinates and carry no physical dimensions, like mass or amplitude ratios, are the best candidates for scheme-independent and/or numerical determinations in quantum and statistical field theories.

In two-dimensional non-linear $O(N)$ σ models some exact results concerning the on-shell (large-distance in the Euclidean space) behavior of the two-point spin-spin Green's function are known. Exact formulas have been presented for the on-shell mass-Λ-parameter ratio [1], and for the constant λ_1 of the $O(3)$ σ model [2], which is defined starting from a parametrization of the large-momentum asymptotic behaviour of the on-shell renormalized two-point correlation function. No exact off-shell results are known. In this letter we study the relation between on-shell and zero-momentum quantities related to the two-point Green's function. This will allow, by using the above-mentioned exact on-shell results, to determine very accurately (the error being of order 10^{-4}) the asymptotic behavior (i.e. for $\beta \to \infty$) of the non-perturbative quantities which parametrize the behavior of the two-point Green's function at small momentum, such as the second-moment mass and the magnetic susceptibility.

For the sake of generality, let us discuss the general $O(N)$ model on a square lattice with nearest-neighbor action

$$S = -N\beta \sum_{x,\mu} \vec{s}(x) \cdot \vec{s}(x+\mu), \tag{1}$$

where $\vec{s}(x) \cdot \vec{s}(x) = 1$. We have introduced a rescaled inverse-temperature β, and we shall use the short-hand

$$\alpha = \frac{N-2}{2\pi N\beta}. \tag{2}$$

We consider the Fourier transform of the bare spin-spin two-point function

$$G(p;\beta) = \sum_x e^{ipx} \langle \vec{s}(x) \cdot \vec{s}(0) \rangle. \tag{3}$$

In the continuum limit and in the large (euclidean) momentum regime a standard one-loop calculation gives

$$G(p;\beta) \underset{p\to\infty}{\longrightarrow} 2\pi \frac{N-1}{N-2} \frac{\alpha^{\frac{N-1}{N-2}}}{p^2} \overline{\alpha}(p)^{-\frac{1}{N-2}}, \tag{4}$$

here $\overline{\alpha}(p)$ denotes the (one-loop) running coupling constant

$$\overline{\alpha}(p) = \frac{\alpha}{1+\alpha \ln ap}, \tag{5}$$

where a is the lattice spacing. The corresponding renormalized Green function $G_R(p,M)$ defined by an on-shell renormalization condition is obtained by requiring

$$G_R(p;M) \underset{p^2+M^2\to 0}{\longrightarrow} \frac{1}{p^2+M^2} \tag{6}$$

where M is the physical mass-gap. The renormalized correlation function can then be parametrized in the large-momentum regime as

$$G_R(p;M) \underset{p\to\infty}{\longrightarrow} \frac{\lambda_1(N)}{p^2}\overline{\alpha}(p)^{-\frac{1}{N-2}}. \tag{7}$$

Notice that Eq. (7) is the definition of $\lambda_1(N)$. This constant depends crucially on the renormalization condition one adopts. In turn the bare two-point function (3) depends explicitly on the coupling. In the low-momentum (large-distance) regime and in the scaling region it can be parametrized by

$$G(p;\beta) \underset{p^2+M^2\to 0}{\longrightarrow} \frac{Z(\beta)}{p^2+M(\beta)^2}. \tag{8}$$

Renormalization-group arguments lead to the following expression for the (universal) asymptotic behavior of the mass-gap:

$$M(\beta) = \frac{1}{a}R(N)\alpha^{-\frac{1}{N-2}}e^{-\frac{1}{\alpha}}\left[1+O(\alpha)\right], \tag{9}$$

and of the on-shell renormalization constant

$$Z(\beta) = C(N)\alpha^{\frac{N-1}{N-2}}\left[1+O(\alpha)\right]. \tag{10}$$

The constant $R(N)$ is not universal and it can be easily computed from the exact result of Ref. [1] by calculating the appropriate Λ-parameter ratio, which can be obtained by a simple one-loop calculation. For the standard nearest-neighbor action on the square lattice one finds

$$R(N) = \left(\frac{8}{e}\right)^{\frac{1}{N-2}}\Gamma\left(1+\tfrac{1}{N-2}\right)^{-1}e^{\frac{\pi}{2(N-2)}}\sqrt{32}. \tag{11}$$

The constant $C(N)$ is universal, that is independent of the lattice regularization. By using Eqs. (4), (7), (8) and (9), it can be put in relation with $\lambda_1(N)$:

$$\lambda_1(N) = 2\pi\frac{N-1}{N-2}\frac{1}{C(N)}. \tag{12}$$

For $N=3$, using the exact result of Ref. [2], i.e.

$$\lambda_1(3) = \frac{4}{3\pi^2}, \tag{13}$$

we get

$$C(3) = 3\pi^3. \tag{14}$$

The main purpose of the present letter is to compute the relation between $R(N)$ and $C(N)$ and the corresponding zero-momentum quantities, which are much easier to compute

in Monte Carlo lattice simulations. We will obtain results which are quite accurate, although not exact.

Typical lattice calculations lead to estimates of moments of the two-point function:

$$m_{2i} \equiv \sum_x (x^2)^i \langle \vec{s}(0) \cdot \vec{s}(x) \rangle. \tag{15}$$

In particular $\chi \equiv m_0$. If we now parametrize the function $G(p;\beta)$ around $p = 0$ by

$$G(p;\beta) \approx \frac{Z_G(\beta)}{p^2 + M_G^2(\beta)} \tag{16}$$

(M_G is by definition the inverse of the so-called second-moment correlation length), we obtain the relationships

$$M_G^2 = \frac{4m_0}{m_2}, \tag{17}$$

$$Z_G = \chi M_G^2 = \frac{4m_0^2}{m_2}. \tag{18}$$

Standard renormalization-group arguments lead again to

$$M_G(\beta) = R_G(N) \alpha^{-\frac{1}{N-2}} e^{-\frac{1}{\alpha}} \left[1 + O(\alpha)\right], \tag{19}$$

$$Z_G(\beta) = C_G(N) \alpha^{\frac{N-1}{N-2}} \left[1 + O(\alpha)\right]. \tag{20}$$

$C_G(N)$ and $R_G(N)$ differ from the corresponding quantities $C(N)$ and $R(N)$ (except at $N = \infty$ where the theory is Gaussian). In order to calculate for these two quantities, we will thus investigate the dimensionless ratios

$$S_M = \lim_{\alpha \to 0} \frac{M^2}{M_G^2}, \qquad S_Z = \lim_{\alpha \to 0} \frac{Z^{-1}}{Z_G^{-1}}. \tag{21}$$

These quantities are not exactly known and we will provide here rather accurate strong-coupling estimates, supported by a $1/N$ analysis.

Strong-coupling estimates of dimensionless RG invariant quantities. As shown in Ref. [3], strong-coupling analysis may provide quite accurate continuum-limit estimates when applied directly to dimensionless renormalization-group invariant ratios of physical quantities. The basic idea is that any dimensionless renormalization-group invariant quantity $R(\beta)$ behaves, for sufficiently large β, as

$$R(\beta) - R^* \sim M(\beta)^2, \tag{22}$$

where R^* is its fixed point (continuum) value and $M(\beta)$ goes to zero for $\beta \to \infty$. Hence a reasonable estimate of R^* may be obtained at the values of β corresponding to large but finite correlation lengths, where the function $R(\beta)$ flattens. This is essentially the same idea underlying Monte Carlo studies of asymptotically free theories, based on the identification of the so-called scaling region. Strong-coupling estimates of physical quantities may be obtained

by evaluating approximants of their strong-coupling series at values of β corresponding to reasonably large correlation lengths, e.g. $\xi \gtrsim 10$. Scaling is then checked by observing the stability of the results varying β.

In a strong-coupling analysis it is crucial to search for improved estimators of the quantities at hand, because better estimators can greatly improve the stability of the extrapolation to the critical point. Our search for optimal estimators was guided by the large-N limit of lattice $O(N)$ σ models which is a Gaussian theory. We chose estimators which are perfect for $N=\infty$, i.e. do not present off-critical corrections to their critical value.

On the lattice, in the absence of a strict rotation invariance, one may define different estimators of the mass-gap M having the same critical limit. On the square lattice one may consider μ obtained by the long-distance behavior of the side wall-wall correlation constructed with $G(x)$, or equivalently the solution of the equation $G^{-1}(i\mu,0;\beta)=0$. At a finite order q of the strong-coupling expansion, the wall-wall spin-spin correlation function $G_w(z)$ at a distance larger than $q/3$ exponentiates exactly, i.e. for $|z|>q/3$ it can be written as

$$G_w(z;\beta) = A(\beta) e^{-\mu(\beta)|z|}. \tag{23}$$

In the context of a strong-coupling analysis, it is convenient to use another estimator of the mass-gap derived from $\mu(\beta)$ [3]:

$$M_{\rm s}^2(\beta) = 2\left(\cosh \mu(\beta) - 1\right). \tag{24}$$

Moreover, by comparison with Gaussian model, we consider the following estimator of Z

$$Z_s(\beta) = 2A(\beta)\sinh\mu(\beta). \tag{25}$$

In practice, when the strong-coupling expansion of $G(x;\beta)$ is known to order q, $M_s^2(\beta)$ can be determined up to about $2q/3$ orders [4], and the same precision is therefore achieved in the determination of $Z_s(\beta)$. Estimators of the zero-momentum mass M_G and renormalization constant Z_G can be easily extracted from Eqs. (17) and (18). So in order to estimate S_M and S_Z one should study the continuum limit of the ratios

$$\overline{S}_M(\beta) \equiv \frac{M_s^2}{M_G^2}, \qquad \overline{S}_Z(\beta) \equiv \frac{Z_G}{Z_s}. \tag{26}$$

In the large-N limit $\overline{S}_M(\beta) = \overline{S}_Z(\beta) = 1$ independently of β. Quantities having the same properties of M_s^2 and Z_s can be conceived also on the honeycomb and triangular lattices [3], thus leading to analogous definitions of $\overline{S}_M(\beta)$ and $\overline{S}_Z(\beta)$.

Analysis of the strong-coupling series. We have analyzed the strong-coupling series of $\overline{S}_M$ and $\overline{S}_Z$ on the square lattice, where the available series [3] are of the form $1+\beta^6\sum_{i=0}^{10} a_i\beta^i$ in both cases. An analogous analysis has been performed on the honeycomb and triangular lattices within their nearest-neighbor formulations using the available series of the two-point function [3]. The analysis of strong-coupling series calculated on different lattices offers a possibility of testing universality. On the other side, once universality is assumed, it represents a further check for possible systematic errors, whose estimate is

usually a difficult task in strong-coupling extrapolation methods such as those based on Padé approximants and their generalizations.

In Table I we present some details of the results obtained on the square lattice. There we report estimates of $\overline{S}_M$ and $\overline{S}_Z$ at various values of β, where the correlation length is reasonably large. Such estimates of $\overline{S}_M$ and $\overline{S}_Z$ have been obtained by resumming the strong-coupling series of $\left(\overline{S}_M - 1\right)/\beta^6$ and $\left(\overline{S}_Z - 1\right)/\beta^6$ by Dlog-Padé approximants (DPA's), in which the standard Padé resummation is applied to the series of the logarithmic derivative, and then the original quantity is reconstructed. This method of resummation turned out to give the most stable results (we also tried simple Padé approximants and first order integral approximants). For a nth order series, we considered $[l/m]$ DPA's having

$$l + m + 1 \geq n - 2, \qquad l, m \geq \tfrac{n}{2} - 2. \tag{27}$$

As estimate at a given $\bar{\beta}$ we took the average of the values of the non-defective approximants constructed using all available terms of the series. As an indicative error we considered the square root of the variance around the estimate of the results from all non-defective approximants specified above. This quantity should give an idea of the spread of the results from different approximants. Approximants are considered defective when they present spurious singularities close to the real axis for $\mathrm{Re}\beta \lesssim \bar{\beta}$.

The precision of the results is satisfactory even for values of β where the correlation length is quite large. Furthermore scaling is well verified. Then, assuming scaling, we extracted estimates of the corresponding continuum limit, which are reported in Table II. There we report results obtained on the square, honeycomb and triangular lattices, and for several values of N ($N = 3, 8, 16$, where the last two large values of N has been considered in order to make a comparison with the large-N analysis, see later). Errors represent a rough estimate of the uncertainty, which is quite small. Universality among different lattice formulations is well verified. Our final estimates for $N = 3$ are

$$S_M = 0.9987(2), \qquad S_Z = 1.0025(4). \tag{28}$$

There are some estimates of S_M obtained by high-statistics Monte Carlo simulations that are worth being mentioned for comparison. Monte Carlo simulations at $N = 3$ [5] gave $S_M = 0.9988(16)$ at $\beta = 1.7/3 = 0.5666...$ ($\xi \simeq 35$), and $S_M = 0.9982(18)$ at $\beta = 0.6$ ($\xi \simeq 65$), leading to the estimate $S_M = 0.9985(12)$. From the data of Ref. [6] one derives $S_M = 0.996(2)$ for $N = 3$ and $S_M = 0.9978(8)$ for $N = 8$. These numbers compare well with our strong-coupling calculations, which appear to be much more precise.

The two-point Green's function at small momentum. The fact that both $S_M \approx 1$ and $S_Z \approx 1$ should not come as a surprise. Indeed it was shown in Ref. [3] that in the region $p^2 \lesssim M_G^2$ the spin-spin two-point function is essentially Gaussian with very small corrections. The inverse two-point function can be studied around $p^2 = 0$ by expanding it in powers of p^2:

$$G^{-1}(p) = Z_G^{-1} M_G^2 \left[1 + \frac{p^2}{M_G^2} + \sum_{i=2}^{\infty} c_i \left(\frac{p^2}{M_G^2} \right)^i \right]. \tag{29}$$

Analysis based on various approaches have shown that in two- and three-dimensional $O(N)$ models the following relations hold [3,7,8]

$$c_2 \ll 1, \qquad c_i \ll c_2 \quad \text{for} \quad i > 2. \tag{30}$$

Then neglecting all c_i, $i \geq 3$ and terms of order c_2^2 one may write the following expression for the inverse two-point function

$$G^{-1}(p) \approx Z_G^{-1} M_G^2 \left[1 + c_2 + \frac{p^2}{M_G^2}\right] \left[1 - c_2 + c_2 \frac{p^2}{M_G^2}\right], \tag{31}$$

which should give a good approximation of the two-point Green's function in the region $|p^2| \lesssim M_G^2$. As a consequence one obtains the following approximate relations

$$S_M \simeq 1 + c_2, \tag{32}$$

$$S_Z \simeq 1 - 2c_2. \tag{33}$$

In order to have a check of Eqs. (32) and (33), we have estimated c_2 by a strong-coupling analysis. Again guided by the large-N limit, we considered the following estimator of c_2

$$\overline{c}_2(\beta) = 1 - \frac{m_4}{64 m_0} M_G^4 + \frac{1}{16} M_G^2. \tag{34}$$

In the continuum limit $\overline{c}_2(\beta) \to c_2$. In the large-$N$ limit $\overline{c}_2(\beta) = c_2 = 0$ independently of β for all square, honeycomb and triangular lattices. We mention that on the square lattice, where $G(x)$ has been calculated up to 21st order [3], the available series of $\overline{c}_2$ is of the form $\beta^4 \sum_{i=0}^{15} a_i \beta^i$. Details of the analysis of $\overline{c}_2$ on the square lattice can be found in Table I. Final estimates are reported in Table II, where also results obtained on the honeycomb and triangular lattices are reported. Numerically in the case of $O(3)$ we obtained $c_2 \simeq -0.0012$, which nicely confirms Eqs. (32) and (33).

$1/N$ **expansion.** We have also evaluated S_M and S_Z in the context of the $1/N$ expansion, which was found to be a fairly accurate approach to the evaluation of amplitude ratios in two-dimensional $O(N)$ models for $N > 2$. An analytic computation of the two-point function in the region $p^2 + M^2 \approx 0$ leads to the following result:

$$G^{-1}(p;\beta) \approx \frac{p^2 + M^2}{2\pi\alpha} \left[1 + \frac{1}{N}\left(\log\frac{4}{\pi\alpha} - \gamma_E - 3\right) + O(1/N^2)\right], \tag{35}$$

and therefore

$$\lambda_1(N) = 1 + \frac{1}{N}\left(\log\frac{4}{\pi} + \gamma_E - 2\right) + O(1/N^2). \tag{36}$$

By evaluating the $O(1/N)$ contribution to the on-shell-renormalized self-energy in the region around $p = 0$, one finds

$$S_M = 1 - \frac{0.00645105}{N} + O\left(\frac{1}{N^2}\right), \tag{37}$$

$$S_Z = 1 + \frac{0.01317046}{N} + O\left(\frac{1}{N^2}\right), \tag{38}$$

$$c_2 = -\frac{0.00619816}{N} + O\left(\frac{1}{N^2}\right), \tag{39}$$

$$c_3 = \frac{0.00023845}{N} + O\left(\frac{1}{N^2}\right), \tag{40}$$

$$c_4 = -\frac{0.00001344}{N} + O\left(\frac{1}{N^2}\right), \tag{41}$$

etc... The coefficients of the $O(1/N)$ terms show consistency with Eqs. (32) and (33) within errors of order 10^{-4}. Inclusion of $c_3 \approx 2.4 \cdot 10^{-4}/N$ would squeeze the error to $O(10^{-5})$. Strong-coupling estimates reported in Table II clearly approach the large-N asymptotic regime predicted by the above equations. In particular quantitative agreement (within the uncertainty of our strong-coupling calculations) is found at $N = 16$. This represents a further check of the analysis employed in order to get strong-coupling estimates in the continuum limit.

It is worth mentioning that c_2, S_M and S_Z have been also calculated to $O(\epsilon^3)$ within the ϕ^4 formulation of $O(N)$ models in $4-\epsilon$ dimensions [8]. The approximate relations (32) and (33) are confirmed even in the ϵ-expansion, whose validity should not be related to the specific value of N. Furthermore a semi-quantitative comparison, inserting the value $\epsilon = 2$ in the ϵ-expansion formulae, provides the correct order of magnititude.

Conclusions. Putting together the exact formulas (11) and (14) and our strong-coupling estimates of the ratios S_M and S_Z, we arrive at the following results

$$R_G(3) = \frac{R(3)}{\sqrt{S_M}} = 80.139(8), \tag{42}$$

$$C_G(3) = C(3) \times S_Z = 93.25(3). \tag{43}$$

For comparison we mention the existing Monte Carlo results concerning the ratio $C_G(3)/R_G(3)^2$. Ref. [9] quotes $C_G(3)/R_G(3)^2 = 0.0146(11)$, which has been obtained by employing finite-size-scaling based techniques allowing to reach correlation lengths up to $O(10^5)$. Ref. [6] quotes $C_G(3)/R_G(3)^2 = 0.0138(2)$, which has been obtained by standard Monte Carlo simulations up to $\xi \simeq 130$, and where the error is just statistical. An attempt to estimate the systematic error due to violations of asymptotic scaling would give the number 0.0138(2)(7)[1], where the second number within brackets is the systematic error estimated by us. These numbers compare very well with our corresponding estimate derived from Eqs. (42) and (43), i.e. $C_G(3)/R_G(3)^2 = 0.01452(5)$.

[1] In Ref. [6] the estimate of the ratio $C_G(3)/R_G(3)^2$ has been obtained by using the Symanzik tree-improved action and considering the so-called energy scheme. In order to estimate the systematic error due to violations of asymptotic scaling, we have considered the difference between the results obtained by using two-loop and three-loop formulas in the fit of Monta Carlo data. Two-loop and three-loop formulas lead to 0.0145(2) and 0.0138(2) respectively.

REFERENCES

[1] P. Hasenfratz, M. Maggiore, and F. Niedermayer, Phys. Lett. **245B**, 522 (1990); P. Hasenfratz and F. Niedermayer, Phys. Lett. **245B**, 529 (1990).

[2] J. Balog and M. Niedermaier, hep-lat/9612039.

[3] M. Campostrini, A. Pelissetto, P. Rossi and E. Vicari, Phys. Rev. **D 54**, 1782 (1996).

[4] M. Campostrini, P. Rossi and E. Vicari, Phys. Rev. **D 52**, 358 (1995).

[5] S. Meyer, unpublished, (1995).

[6] B. Allés, A. Buonanno and G. Cella, hep-lat/9701001.

[7] M. Campostrini, A. Pelissetto, P. Rossi and E. Vicari, Phys. Rev. **B 54**, 7301 (1996).

[8] M. Campostrini, A. Pelissetto, P. Rossi and E. Vicari, cond-mat/9612164, and in preparation.

[9] S. Caracciolo, R. G. Edwards, T. Mendes, A. Pelissetto, and A. D. Sokal, Nucl. Phys. (Proc. Suppl.) **B** 47, 763 (1996).

TABLES

TABLE I. Estimates of $\overline{S}_M$, $\overline{S}_Z$ and $\overline{c}_2$ obtained from DPA's of the strong-coupling series on the square lattice evaluated at various values of β where the correlation length ξ is reasonably large. For example at $N=3$: $\xi(\beta=0.45)\simeq 8$, $\xi(\beta=0.50)\simeq 11$, $\xi(\beta=0.55)\simeq 25$, and $\xi(\beta=0.60)\simeq 65$; at $N=8$: $\xi(\beta=0.50)\simeq 5$, $\xi(\beta=0.55)\simeq 8$, and $\xi(\beta=0.60)\simeq 12$.

N		$\beta=0.45$	$\beta=0.50$	$\beta=0.55$	$\beta=0.60$
3	$(\overline{S}_M-1)\times 10^3$	$-1.01(3)$	$-1.2(1)$	$-1.1(2)$	$-0.9(3)$
	$(\overline{S}_Z-1)\times 10^3$	2.04(6)	2.4(2)	2.2(4)	1.4(7)
	$\overline{c}_2\times 10^3$	$-1.16(4)$	$-1.3(2)$	$-1.4(3)$	$-1.3(4)$
8	$(\overline{S}_M-1)\times 10^3$	$-0.54(1)$	$-0.59(2)$	$-0.57(5)$	$-0.5(1)$
	$(\overline{S}_Z-1)\times 10^3$	1.11(3)	1.2(1)	1.2(1)	1.1(2)
	$\overline{c}_2\times 10^3$	$-0.58(2)$	$-0.63(5)$	$-0.6(1)$	$-0.6(2)$
16	$(\overline{S}_M-1)\times 10^3$	$-0.32(4)$	$-0.35(6)$	$-0.4(1)$	$-0.4(2)$
	$(\overline{S}_Z-1)\times 10^3$	0.7(1)	0.7(1)	0.8(2)	0.9(4)
	$\overline{c}_2\times 10^3$	$-0.31(1)$	$-0.34(3)$	$-0.35(6)$	$-0.35(10)$

TABLE II. We report estimates of S_M, S_Z and c_2 from the strong-coupling expansion on the square, honeycomb and triangular lattice, and $1/N$ expansion of the continuum formulation of the non-linear $O(N)$ σ model. Final strong-coupling estimates are taken at β-values corresponding to $\xi\simeq 10$.

N		S_M	S_Z	c_2
3	square	0.9988(2)	1.0024(4)	$-1.3(2)\times 10^{-3}$
	honeycomb	0.9986(3)	1.0027(4)	$-1.2(2)\times 10^{-3}$
	triangular	0.9985(5)	1.003(1)	$-1.2(3)\times 10^{-3}$
	$O(1/N)$	0.9978	1.0044	-2.07×10^{-3}
8	square	0.99943(5)	1.0012(1)	$-0.6(1)\times 10^{-3}$
	honeycomb	0.9994(1)	1.0011(2)	$-0.7(1)\times 10^{-3}$
	$O(1/N)$	0.99919	1.00164	-0.77×10^{-3}
16	square	0.9996(1)	1.0008(2)	$-0.35(5)\times 10^{-3}$
	honeycomb	0.9997(1)	1.0006(1)	$-0.36(3)\times 10^{-3}$
	$O(1/N)$	0.99960	1.00082	-0.39×10^{-3}